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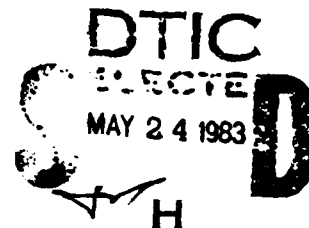## DEFENCE SCIENCE AND TECHNOLOGY ORGANISATION

AERONAUTICAL RESEARCH LABORATORIES

MELBOURNE, VICTORIA

Aerodynamics Technical Memorandum 344

ESTIMATION PROGRAMS FOR DYNAMIC FLIGHT TEST DATA ANALYSIS

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ESTIMATION PROGRAMS FOR DYNAMIC FLIGHT TEST
DATA ANALYSIS

by

R.A. FEIK

SUMMARY

A brief overview of the application of System Identification
methodology to the analysis of dynamic flight test data is provided,
and the advantages of this approach summarised. Parameter estimation
is presented as an important element in the overall process and a
number of estimation programs, available on the ARL computer, are
described. The advantages and disadvantages of the various procedures,
including least squares, maximum likelihood and Kalman filter
techniques, are assessed. Suggestions are made on factors to consider
in choosing the most suitable method, depending on measurements
available, noise levels and model structure.

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CONTENTS

1. INTRODUCTION

The application of system identification methodology to the extraction of aerodynamic information from flight test data dates from the mid sixties, and has been made possible by the development of fast digital computers. Although flight test conditions pose a difficult environment for making accurate measurements, considerable success has been achieved. Much of this success can be attributed to the availability of good quality data acquisition systems and to the existence of well defined linear models for the description of the aircraft motions.

The advantages offered by these techniques include improved accuracy and decrease in flight test time required. Many parameters are obtained from a single manoeuvre, unlike older methods which often require a specific manoeuvre for a given parameter on a one-for-one basis. Manoeuvres not previously amenable to analysis can be processed without difficulty.

These methods are now used extensively overseas and are standard for stability and control testing in many establishments. Their application to performance flight testing is still under development due to more stringent accuracy requirements. Developments are also being aimed at testing in non-linear flight regimes, particularly high angle-of-attack manoeuvring.

At ARL, the Aircraft Behaviour Studies - Fixed Wing Group of Aerodynamics Division commenced activity in this area in the mid seventies. The principal aim has been to aid in the development and validation of flight dynamic mathematical models of aircraft such as the Mirage and the F111C. The Group has acquired or developed a number of parameter and state estimation computer programs and obtained experience in their use and application to flight test data analysis. It is the purpose of this Memo to describe these programs, including their advantages and disadvantages, and to point to areas of application. By doing so it is hoped that they may be brought to the attention of a wider range of potential users.

It should be noted that the term "system identification" as used here refers to a complete procedure and includes consideration of a full range of elements such as manoeuvre shape, instrumentation accuracy, data acquisition rates, mathematical model definition and so on, as summarised in Figure 1. Although the estimation programs described in this Memo are an essential part of the procedure, the quality of the results obtained will depend to a considerable extent on proper consideration being given to each of the other elements.

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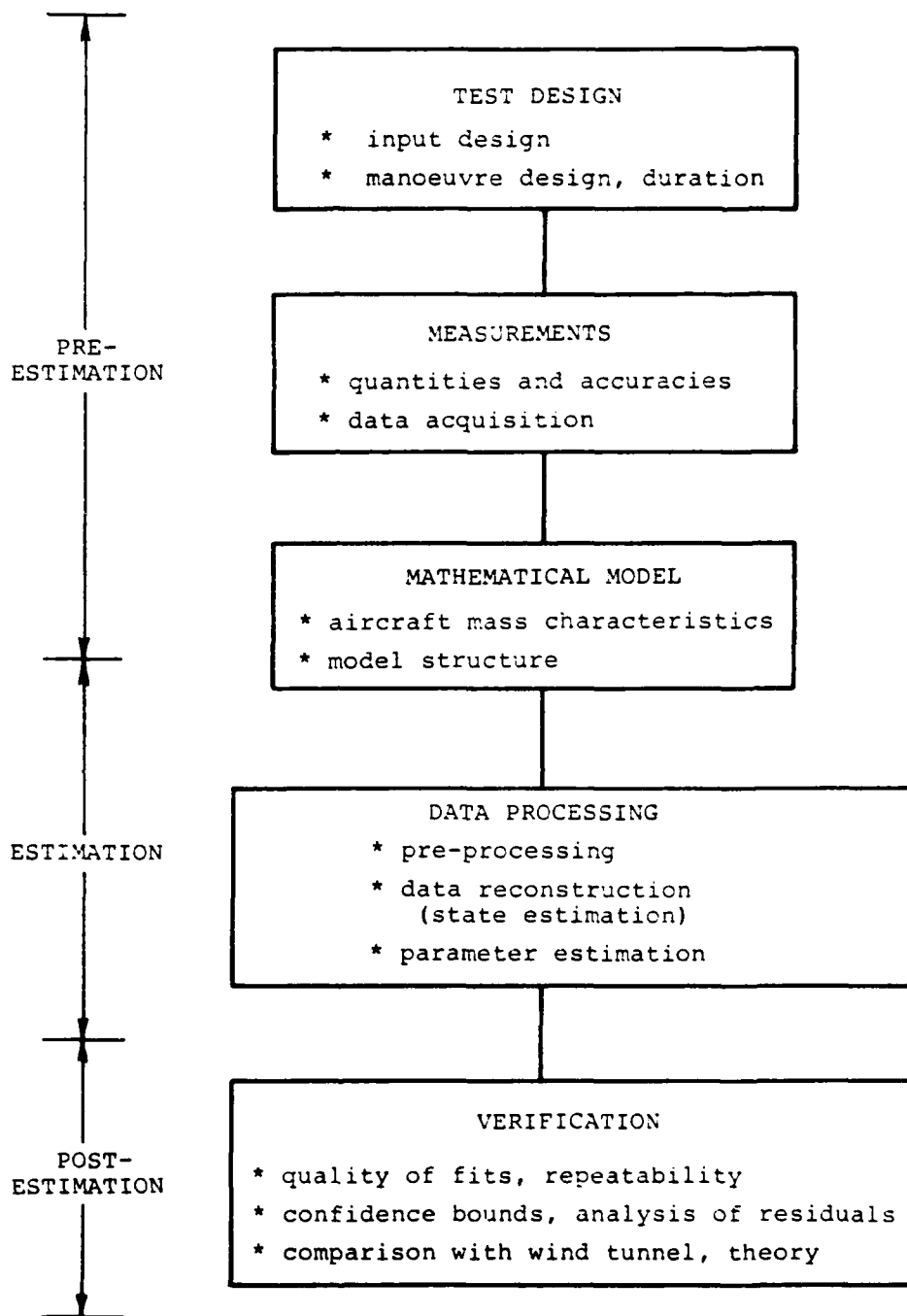PRE-ESTIMATION

ESTIMATION

POST-ESTIMATION

FIG. 1   INTEGRATED SYSTEM IDENTIFICATION PROCEDURE

2.   PARAMETER ESTIMATION

Before proceeding to describe the programs, some preliminary remarks may be useful. The parameter estimation problem can be described in general terms as follows: The system investigated is assumed to be modelled by a set of dynamic equations containing unknown parameters. For example the equations describing aircraft dynamics contain the aerodynamic derivatives as unknowns. The system is excited by a suitable input and measurements made of the input and actual system response. The model response is calculated using the same input and the model parameters are adjusted systematically so that the model response matches the measured response as closely as possible.

In reality, random excitation of the system due to unmeasured sources will also be present. A typical example of this for aircraft is atmospheric turbulence. This is referred to as process noise (also state noise or input noise). Thus a general description of the actual system can be assumed to be given by a system of differential equations of the form

$$\dot{x}(t) = f\,(x(t),\ u(t),\ \xi_{true}) + w(t) \qquad (1)$$

where f is in general a non-linear function and

x is the state vector

u is the input vector

ξ is the vector of system parameters

w is the process noise vector.

Further, measurements of the response inevitably contain errors, both of a deterministic nature, such as instrument bias and scale factor errors, and random measurement errors. Thus the response measurements, $z(i)$, can be assumed to be described by

$$z(i) = g\,(x(i),\ u(i),\ \xi_{true}) + v(i) \qquad (2)$$

where g is in general a non-linear function

   v is the measurement noise vector

   i is a discrete time point at which the measurements were made.

The unknown parameter vector, ξ, will include both
system parameters and parameters modelling instrument systematic
errors. Unless the latter are adequately modelled, results may
be seriously in error. This raises the question of modelling real
systems. Physical systems are seldom described by simple dynamic
models and this implies the existence of modelling errors.
Commonly, modelling errors are simply treated as process and/or
measurement noise in spite of the fact that they may be deterministic
rather than random.

For the aircraft case, well-defined linear models
have been established for small disturbance motions about an
equilibrium state. However, for large manoeuvres or for high
angles of attack it may be necessary to consider non-linear
representations of the aircraft motions.

In choosing a parameter estimation procedure
consideration needs to be given to the level of process and
measurement noise present, whether a linear model is an adequate
representation of the system or whether a non-linear, perhaps ill-
defined, model needs to be investigated. The programs described
in the next section offer a choice in line with these considerations
and also depending on what measurements are available.

3.   PROGRAMS

The five programs to be described are presented
roughly in order of increasing complexity. The underlying methods
and their theoretical properties are broadly discussed and their
range of applicability outlined, including whether process and/or
measurement noise are catered for and whether a linear model is
assumed. For more detailed information appropriate references are
provided for the reader to consult. The advantages and disadvantages
of each method are also discussed, in the light of experience obtained
both at ARL and elsewhere. The programs themselves, including input/
output facilities, are only described in fairly broad terms and the
potential user will need to consult the references and/or the present
author for more specific information about their operation on the
ARL computer.

3.1 Equation Error - Least Squares

To illustrate this approach consider the following
example: It is assumed that the aerodynamic force, Z acting on the
aircraft in the z-direction can be represented by

$$a_z = \frac{Z}{m} = b_0 + b_1 \cdot \alpha + b_2 \cdot q + b_3 \cdot \delta_e \tag{3}$$

$$+ b_4 \cdot \alpha^2 + b_5 \cdot \alpha\delta_e + b_6 \cdot \delta_e{}^2$$

where the coefficients b_0, b_1 .. etc. (related to the aerodynamic derivatives) are required to be estimated, and measured records are available for the independent variable, a_z (acceleration in the z-direction) and the dependent variables α (angle of attack), q (pitch rate) and δ_e (elevation angle). Note that the assumed model, Equation (3), contains terms non-linear in α and δ_e. By substituting values for measured quantities at each time point i, the equation can be written

$$a_z(i) - b_0 - b_1 \cdot \alpha(i) - b_2 \cdot q(i) - \ldots - b_6 \cdot \delta_e{}^2(i) = \varepsilon(i) \quad (4)$$

where, because Equation (3) is only an approximation, the right hand side of Equation (4) is referred to as the Equation Error, and can account for measurement noise and/or modelling errors. Equation (4) is an example of the more general form

$$y = X \, \xi + \varepsilon \qquad (5)$$

where, referring to Equation (4), for N time points

$$y = [a_z(1), \ a_z(2), \ \text{--------} \ a_z(N)]^T$$

$$\varepsilon = [\varepsilon(1), \ \varepsilon(2), \ \text{--------} \ \varepsilon(N)]^T$$

$$X = \begin{bmatrix} 1 & \alpha(1), & q(1) & \text{--------} & \delta_e{}^2(1) \\ 1 & \alpha(2), & q(2) & \text{--------} & \delta_e{}^2(2) \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ . & . & . & & . \\ 1 & \alpha(N), & q(N) & \text{--------} & \delta_e{}^2(N) \end{bmatrix}$$

and

$$\xi = [b_0, \ b_1, \ \text{--------} \ b_6]^T$$

The least squares estimate for the unknown parameters, ξ, given the measurements of y and X, is

$$\hat{\xi} = (X^TX)^{-1} X^Ty \tag{6}$$

If $\varepsilon(i)$ are zero mean and independent with variance σ^2, then the least squares estimate is a Best Linear Unbiased Estimator with covariance of the estimates given by

$$E[(\hat{\xi}-\xi)(\hat{\xi}-\xi)^T] = (X^TX)^{-1} \sigma^2 \tag{7}$$

and the value of σ^2 can be estimated from the sum of squares of the residuals i.e.

$$\sigma^2 = \frac{1}{(N-p)}(y - X\hat{\xi})^T(y - X\hat{\xi}) \tag{8}$$

with p being the dimension of the parameter vector, ξ. Further details can be found in References 1 and 2.

Reference 2 demonstrates the important reservation that the estimates given by Equation (6) will be biased unless X (the matrix of dependent variables) is measured without error or, as mentioned above, the Equation Error, $\varepsilon(i)$, is zero mean and white. On the other hand, process noise does not lead to biased estimates. Process noise, in this case, is equivalent to a model of the form

$$y_T = X_T \xi + w \tag{9}$$

where the subscript T denotes true values and w is the process noise. However, process noise, as well as measurement noise on the independent or dependent variables, y or X, does lead to degradations in accuracy as measured by the covariance of the estimates, Equation (7).

Despite these deficiencies, the Equation Error method is often used for estimating aerodynamics from flight test data and can be expected to lead to good results when measurement noise levels are not too high. Its main advantages are its simplicity, easy application to any linear or non-linear model (Equation (3)) and its modest computing demands. A practical requirement is that measurements of all the variables in y and X must be available. If one of the measurement channels drops out then the method breaks down.

A least squares program LSPROG has been developed
for the ARL DEC System 10 computer. It has been written using a
robust algorithm based on the Householder transformation, which has
advantages when the equations are ill-conditioned (Ref. 1). A
direct solution using matrix inversion is also included. The inputs
required are the dimensions of the X matrix and tabulation of the
y and X measurements at each time point. The program is interactive
and will calculate specified confidence intervals on request.

3.1.1 Program Options

The least squares program LSPROG provides two options
which have been found useful when the functional form of the fitted
model (Equation (3)) is uncertain. The first option allows the user
to try various constraints on the solutions. This is done by
specifying a set of constraint relations of the form

$$L \, \xi \; = C \tag{10}$$

where L is a matrix of dimension s by p, s being
the number of constraints and p the dimension of the parameter vector,
ξ, while C is a vector of dimension s. The value of s and the
elements of L and C are provided interactively by the user. As a
simple example, one or more elements of ξ could be set to zero.
Further details of the method can be found in Reference 1, Chapter
2.

The second option is a Backward Elimination Procedure
which, starting from a given model such as that in Equation (3),
will systematically eliminate the least significant terms until
further eliminations become statistically unjustifyable according to
a pre-selected level of significance. The final result is a best
Regression solution which retains only those terms found to be
significant at the desired level. The level of significance,
specified interactively by the user, is a measure of the risk of error
i.e. the probability of retaining a term when it should be eliminated
(Type I error in statistical hypothesis testing). Details of the
method can be found in References 3 and 4.

3.2 Output Error - Maximum Likelihood (Linear Systems)

The output error method seeks to minimise the
difference between the measured output and the model output (using the
same input) by suitable choice of the unknown parameters.

For the particular case where the system (e.g. aircraft)
can be described by a set of linear, constant coefficient equations,

and where there are no gust or other unknown disturbances (i.e. no process noise), Equations (1) and (2) can be written as

$$\dot{x} = Ax + Bu \tag{11}$$

$$z(i) = C\,x(i) + D\,u(i) + b + v(i) \tag{12}$$

where matrices A,B,C,D contain parameters (e.g. aerodynamic derivatives) defining the system, b is a constant bias vector, and v is the measurement noise vector. The unknown parameter vector, ξ, contains some or all of the elements of A,B,C and D, the unknown biases b, and the initial conditions.

The Maximum Likelihood (ML) estimates of the unknown parameters are those for which the observed values of the output, z, would be most likely to occur i.e. the conditional probability density $p(z/\xi)$ (the probability of z given ξ) is maximised with respect to ξ. If the measurement noise vectors $v(i)$ are zero mean independent Gaussian vectors then the probability density. also known as the Likelihood function, can be written

$$p(z/\xi) = [2\pi^{m}|R|]^{-\frac{1}{2}N} \exp\left\{-\frac{1}{2}\sum_{i=1}^{N}[\hat{z}(i)-z(i)]^{T} R^{-1}[\hat{z}(i)-z(i)]\right\}$$

$$\tag{13}$$

where $\hat{z}$ refers to the expected value of the observation, z, and is obtained (for no process noise) from Equation (12), neglecting $v(i)$, after simple integration of Equation (11). In Equation (13), R is the mxm measurement noise covariance matrix, with m the dimension of the observation vector, z. Instead of maximising $p(z/\xi)$ it is usual to minimise a cost functional, J, obtained by taking the negative log of $p(z/\xi)$, giving

$$J(\xi,R) = \frac{1}{2} N \log|R| + \frac{1}{2}\sum_{i=1}^{N}[\hat{z}(i)-z(i)]^{T} R^{-1}[\hat{z}(i)-z(i)]$$

$$\tag{14}$$

The M.L. method thus finds a set of parameters, ξ, which minimises the cost function J defined by Equation (14). If R is not known then an estimate for R can be obtained by minimising J with respect to R, which results in the following estimate for the covariance matrix

$$\hat{R} = \frac{1}{N}\sum_{i=1}^{N}[\hat{z}(i)-z(i)][\hat{z}(i)-z(i)]^{T} \tag{15}$$

Estimates of the unknown parameters, ξ, are then
obtained by minimising J with respect to ξ, with $\hat{R}$ from Equation
(15) replacing R. Since J is a non-linear function of ξ, an
iterative technique is required to do this. A common choice is a
modified Newton-Raphson algorithm (Ref. 5). The new value of ξ
thus obtained can be used to calculate a new $\hat{z}(i)$ and hence a
revised estimate of $\hat{R}$ follows from Equation (15). This two-step
procedure can be repeated until convergence is achieved.

The ML estimates are unbiased and consistent i.e.
they converge to the true values for large N. Provided that the
noise vectors are independent, the estimates are also efficient i.e.
the covariance of the estimates approaches the Cramer-Rao lower
bound for large N (Ref. 1). Interpretation of the Cramer-Rao bound
for real flight data is discussed in Reference 6. The Cramer-Rao
bound and hence the covariance of the estimates is readily calcula-
ted as follows

$$E\{(\hat{\xi}-\xi)(\hat{\xi}-\xi)^T\} = \{ \sum_{i=1}^{N} A^T(i)\ \hat{R}^{-1}\ A(i) \}^{-1} \tag{16}$$

where A is the sensitivity matrix whose elements are
the partial derivatives of the elements of z with respect to the
elements of ξ. The calculation of A is also required in the basic
Newton-Raphson algorithm to solve for the unknown parameters, ξ, and
follows from Equations (11) and (12).

The present Output Error (OE) method, unlike the
Equation Error (EE) method of the previous section, can cope with the
loss of a data channel, simply by setting the relevant element of
the weighting matrix, R^{-1} to zero. Since the complete system of
equations is treated simultaneously, information can be passed between
equations so that parameter estimates can still be obtained. Such
a trade-off is not possible with the EE method which treats each
equation independently of the others. Computing time is considerably
longer for the OE method than for the EE approach. However, the
main disadvantages of the OE method as applied here are the limita-
tion to linear models and the degradation of the results in the presence
of process noise. While measurement noise is handled well, process
noise (i.e. unmodelled disturbances or possibly due to modelling errors)
may result in the program not converging or in poor estimates with
large covariances. In this sense it is complementary to the EE
approach which copes well with process noise but not with measurement
noise. The OE method has been widely applied to aircraft testing for
several years and works very well with linear flight regimes and
smooth test conditions. Extensions to handle process noise and applica-
tions to non-linear, but well defined models, are considered in
subsequent sections.

The method described in this section is the basis
of the ARL program NEWTN4, a slightly modified version of the program
described in Reference 5. A detailed description of the program,
including input requirements and output produced, is given in
Reference 5. Briefly, inputs include information defining the system
matrices, weighting matrices and parameters to be estimated as well
as input and response measurement time histories. The outputs
include a convergence history, fit errors and the resulting parameters
and their covariances. The program allows the user to set the
weighting matrix, R^{-1}, thereby fixing the relative worth of each
observation channel (or component of the measurement vector, z).
Examples of its use at ARL are reported in References 7 and 8. Easy
application and generally good performance make the program very
suitable for stability and control flight testing, where a well
established linear model exists for describing aircraft response to
control inputs.

3.2.1 A Priori Option

If information about some elements of the parameter
vector, ξ, is available from other sources (e.g. wind tunnel,
theoretical estimates or previous tests), then it is sometimes
desirable for the estimating algorithm to consider this a priori
information in addition to the new data from the current manoeuvre.
In NEWTN4 this is accomplished by adding to the cost functional of
Equation (14) a quadratic penalty term for departure from the a
priori value

$$\Delta J = \tfrac{1}{2}(\xi-\xi_o)^T \, W(\xi-\xi_o) \tag{17}$$

where ξ_o is the vector of a priori values and W
is the a priori weighting matrix set by the user. Clearly the penalty
ΔJ will be zero when $\xi = \xi_o$ and will increase for any departure
from ξ_o Such an increase would be countered by a decrease in the
rest of the cost functional J (Eqn. 14). Components of the parameter
vector, ξ, will thus only depart from ξ_o if there is sufficient
information in the new data to justify this. If J (Eqn. 14) is
very weakly dependent on a particular parameter then the a priori
option will ensure that its estimated value will not depart far from
the a priori value. This feature can be used to check whether
significant information on a specific parameter is available from
the new data. For example if the covariance of the estimate is
significantly less than that implied by the a priori weight (or
element of the W matrix in Eqn. (17)) then it can be concluded that
the new data contains worthwhile information on that parameter.

However, care must be taken in the use of the a priori option since results will inevitably be biased towards the a priori values, the amount of bias depending on the weighting matrix W. Thus results should be checked with decreasing values of W and, finally, with the a priori option removed.

3.3 Generalised Maximum Likelihood (Linear Systems)

The output error method described in the previous section can be generalised to the case where process noise is present. For a linear system with zero mean, white Gaussian w(t), Equations (1) and (2) can be written

$$\dot{x}(t) = Ax(t) + B u(t) + w(t) \tag{18}$$

$$z(i) = C x(i) + D u(i) + v(i) \tag{19}$$

The Maximum Likelihood approach, as in section 3.2, minimises the negative log of the Likelihood function, resulting in the following cost functional to be minimised:

$$J(\xi,G) = \tfrac{1}{2}N \log|G| + \tfrac{1}{2} \sum_{i=1}^{N} [\hat{z}(i)-z(i)]^T G^{-1} [\hat{z}(i)-z(i)] \tag{20}$$

where $\hat{z}$ is the expected value (or predicted estimate) of z and G is the covariance of the residuals, $\hat{z}(i)-z(i)$. Equation (20) is identical to Equation (14) except that $\hat{z}$ is now computed using a Kalman Filter, which provides an optimal estimate taking into account both process and measurement noise, and R is replaced by G, which includes information on both measurement and process noise covariances. Equation (14) can be viewed as a special case of Equation (20) for zero Kalman gain. Detailed discussion of Kalman Filtering can be found in Reference 9.

The discussion of section 3.2 relating to solution of the minimisation problem, the asymptotic properties of the method and the estimated covariances of the results (Eqn. (16)) applies also to this section provided R is replaced by G and z is understood to be the output of a Kalman Filter.

The computer program MMLE3, described in detail in References 10 and 11, has been implemented on the ARL computer. MMLE3 has evolved from the earlier program of Reference 5 and, apart from the generalisation to handle process noise, has more flexible input/output facilities and inbuilt plotting capabilities, designed to make the task of the user as easy as possible. The program is organised into two levels. At the basic level it consists of a

general maximum likelihood program applicable to any linear system.
At the second level it can be adapted to a particular application
through a set of routines which can be written or modified by the
user for that particular application. Having done this, the program
input does not then have to contain the detailed system specifica-
tions but only those items which change from case to case. In
particular, a set of standard aircraft user routines are provided
for the aircraft longitudinal or lateral stability and control
problem with no turbulence.

The process noise capability of MMLE3 should be
approached with considerable care since this feature guarantees a
good fit between the measured and estimated responses. Therefore
possible modelling problems, which would ordinarily lead to a poor
fit, may not become apparent. Thus it may be advisable for the
inexperienced user to avoid using this capability before gaining
a clear understanding of all aspects of the program and the
particular problem in hand.

MMLE3 also has an a priori option which can be
switched off after a specified number of iterations. Reference 9
recommends that this be done to allow the last few iterations to
run without a priori weighting, and thus achieve an unbiased result.

3.4 Maximum Likelihood (Non-Linear Systems)

With this program the method of section 3.2 is
extended to deal with non-linear models but is restricted to the no
process noise case. Thus the system model is represented by

$$\dot{x}(t) = f(x(t), u(t), \xi) \tag{21}$$

$$z(i) = g(x(i), u(i), \xi) + v(i) \tag{22}$$

The general procedure is identical to that described
in section 3.2. In the present case the expected values of the
output, z, are obtained by integrating Equation (21) and substitu-
ting for $x(i)$ into Equation (22) (neglecting $v(i)$). Since the form
of the non-linear functions, f and g in Equations (21) and (22),
will differ from problem to problem the user must provide subroutines
to carry out these calculations for each specific problem. In
addition, the user must provide for the calculation of the sensitivity
matrix, A, which is required both in the Newton-Raphson solution
algorithm and also in the calculation of the covariance matrix of
the estimates, Equation (16). As noted in section 3.2, the elements
of A are the derivatives of the elements of the output vector, z,
with respect to the elements of the parameter vector, ξ, and their

calculation will be problem dependent. The general properties of
the maximum likelihood method and its advantages and disadvantages
are as discussed in section 3.2.

A computer program which has evolved from the
program of section 3.2 has been written for the ARL computer to
implement the procedure. The problem dependent aspects referred
to above require the user to provide a number of specific sub-
routines to define the system. The structure of the program and
requirements of the user supplied subroutines are described in
Reference 12. Input and output data are also described and are
broadly similar to that outlined in 3.2.

Reference 12 describes a particular application of
the program to the compatibility checking of Flight measurements.
By suitably reformulating the kinematic equations describing the
aircraft motion, instrument systematic errors such as biases and
scale factor errors can be identified, and a consistent set of
'error-free' measurements obtained. At the same time, missing
records can be reconstructed if required. With a complete set of
clean measurements thus available, a second stage of analysis can
proceed to extract aerodynamic parameters and possibly examine
alternative aerodynamic model structures such as that of Equation
(3). The regression program of section 3.1 may be a useful method
of doing this and would be expected to yield good results in the
presence of small measurement errors. The two stage procedure
proposed allows instrument errors to be processed separately from
any consideration of the aerodynamic model or parameters. The
ability to check the compatibility of the measured data is in fact
a useful facility irrespective of the subsequent analysis.

3.5 Extended Kalman Filter

In the most general case, a non-linear model may
be required to describe adequately the system under consideration,
and both process and measurement noise may be present. This case
is summarised by Equations (1) and (2) which are repeated below

$$\dot{x} = f(x(t), u(t), \xi) + w(t) \tag{23}$$

$$z(i) = g(x(i), u(i), \xi) + v(i) \tag{24}$$

The extended Kalman Filter (EKF) is an approximate
filter for non-linear systems, based on first order linearisation
of the state and output equations about the best estimate of the
state at each data point. The filter consists of prediction
equations, where the expected value of the state is predicted one

step ahead, and mesurement update equations, where the measured data
of that time point are used to update the predicted value. Although
the Kalman Filter is a State estimator it can be applied to Parameter
estimation by including the unknown parameters in the state vector.
This is done by augmenting the system state vector, x, with the
parameter vector, ξ, so that the augmented state vector, x_A, becomes

$$x_A = [x, \xi]^T \tag{25}$$

The state equations, in addition to Equation (23),
are

$$\xi = 0 \tag{26}$$

since the unknown parameters, ξ, are assumed to remain
constant with time. Thus, while the parameters remain constant between
measurements, they are updated with the new information at each data
point. Including the parameters as state variables in this way
increases the non-linearities in Equation (23), which is already non-
linear in structure. Successful application of the EKF depends on the
accuracy of the linearisation process. For sufficiently small step
size good results can be obtained, and in this case, since the EKF
estimates are essentially the maximum likelihood estimates, the
properties of the EKF estimator are identical to those mentioned in
section 3.3 (Ref. 2).

The main disadvantage of the method is the need for
information on the noise statistics and for a priori covariances
which are unknown for the parameters. If the a priori values for the
parameters are poor, then the method can fail to converge (Ref. 2).

An EKF program has been developed at ARL and is
described in Reference 13. At present the program is specifically
configured to deal with the compatibility checking problem outlined
in section 3.4. The use of the EKF allows the inclusion of process
noise in the formulation of that problem. Neglect of the process
noise leads to an underestimate of the resulting parameter covariances,
whereas the EKF will calculate these correctly, in theory. A
comparison of the two approaches has been made in Reference 14 where
the theoretical advantage of the EKF was only seen to be achieved
with low noise levels. Further development is in progress.
Application of the ARL EKF program to other non-linear systems would
require some re-writing of those sections of the program which deal
with the problem formulation.

4. CONCLUDING REMARKS

A brief overview of the application of System Identification methodology to the analysis of dynamic flight test data has been provided, showing the parameter estimation program as an important element in the overall process. Advantages of the System Identification approach have been summarised in the Introduction. A number of estimation programs, making use of least squares, maximum likelihood and Kalman filter procedures, are available on the ARL computer and have been described in this Memo. Advantages and disadvantages of each method have been pointed out.

In choosing a program, a number of aspects of the particular problem in hand need to be considered. These include:

(i) the size and structure of the mathematical model, whether well-defined, linear or otherwise,

(ii) the level of process and measurement noise present,

(iii) the measurements available.

Thus, if suitable measurements of all required quantities are available and measurement noise levels are small, the least squares approach (section 3.1) has been shown to yield good results even in the presence of process noise, and has the added advantage that non-linear representations can be assumed. Further, the program provides a capability for comparing different model structures and choosing the most significant.

If a well established, linear model is available, such as that describing aircraft stability and control over a restricted angle of attack range, the use of a maximum likelihood program (e.g. section 3.2) is a good choice and can be used even when some output measurements are not available. In the presence of significant process noise a generalised maximum likelihood scheme (section 3.3) can be used.

For a non-linear model with not all output quantities measured, the non-linear maximum likelihood approach at section 3.4 can be expected to give reasonable results if process noise levels are low. Alternatively, a two stage procedure as outlined in section 3.4 may be preferred especially if the number of unknown parameters is large. With a significant level of process noise this would require the use of a procedure such as the Extended Kalman Filter of section 3.5.

Specific information on the running of these programs on the ARL computer is provided, to a large extent, in the cited references. For additional information the potential user is invited to contact the present author.

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16. Abstract

A brief overview of the application of System Identification methodology to the analysis of dynamic flight test data is provided, and the advantages of this approach summarised. Parameter estimation is presented as an important element in the overall process and a number of estimation programs, available on the ARL computer, are described. The advantages and disadvantages of the various procedures, including least squares, maximum likelihood and Kalman filter techniques, are assessed. Suggestions are made on factors to consider in choosing the most suitable method, depending on measurements available, noise levels and model structure.

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